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Ground-state triply and doubly heavy baryons in a relativistic three-quark model

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Mass spectra of the ground-state baryons consisting of three or two heavy (b or c) and one light (u, d, s) quarks are calculated in the framework of the relativistic quark model and the hyperspherical expansion. The predictions of masses of the triply and doubly heavy baryons are obtained by employing the perturbation theory for the spin-independent and spin-dependent parts of the three-quark Hamiltonian.

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The transition from two-quark bound states to three-quark bound states opens new problems which refer both to the form of the quark interaction in the baryon and the structure of three-quark relativistic wave equation describing this system. In general form they were studied and solved already by many authors [1, 2, 3, 4, 5, 6, 7, 8] (see other references in Ref.[6]). In practice it is important to have an approach which can allow to obtain simple and reliable estimates for the different experimental quantities regarding to the baryon spectroscopy, the production and decay rates. Whereas heavy baryons with one or two heavy quarks were investigated both theoretically and experimentally, the triply heavy baryons $\Omega_{Q_1Q_2Q_3}$ containing $b-$ and $c-$ quarks have not studied so much. The estimate of masses of the lowest-lying (ccc), (ccb), (bbc) and (bbb) states is presented in Refs.[9, 10, 11]. Their production in a $c-$ or $b-$ quark fragmentation is calculated in Refs.[12, 13]. The doubly heavy baryons (Q_1Q_2q) represent a unique part of three-quark systems. Two heavy quarks compose a localized quark nucleus while the light quark moves around this color source at a distance of order ($1/m_q$). This picture leads to the quark-diquark model for doubly heavy baryons which was used in Refs.[14, 15, 16] for the description of the mass spectrum and decay widths. Moreover, relativistic and bound state corrections to the mass spectra of mesons and baryons (in the quark-diquark approximation) and their different decay rates were also considered in the relativistic quark model [14, 15, 16, 17, 18]. Estimates for the masses of baryons containing two heavy quarks have been presented by many authors [4, 19, 20, 21, 22, 23] using different QCD inspired models for the quark interactions. The aim of the present paper is a twofold one. First, we go beyond the scope of the quark-diquark approximation in Refs.[14, 15] treating the total baryon Hamiltonian as a sum of two-quark interactions and using the hyperradial approximation for the ground state triply and doubly heavy baryons. Secondly, we take into account relativistic and bound state corrections of order v^2/c^2 by the perturbation theory. So, the purpose of our new investigation consists in the elaboration of an alternative calculational scheme of the baryon mass spectrum as compared with the earlier performed investigations in Refs.[14, 15] through the use of

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the three-quark approach to the baryon problem formulated in Refs.[19, 20, 21] with the Hamiltonian containing the spin-independent and spin-dependent corrections of order v^2/c^2.

It should be noted, that the theoretical results on the mass spectrum of triply and doubly heavy baryons [5, 6, 7] remain untapped. In the past several years, the SELEX Collaboration has reported the first observation of doubly charmed baryons [24, 25]. But most recently, BaBar Collaboration has reported that they have not founded any evidence of doubly charmed baryons in e^+e^- annihilation [26]. Nevertheless, we can expect that the mass spectra and decay rates of triply and doubly heavy baryons will be measured before long. This gives additional grounds for new theoretical investigations of triply and doubly heavy baryon properties.

In order to describe the mass spectra of baryons the different three-quark Hamiltonians are used [7, 14, 19, 27]. The effective Hamiltonian of Refs.[19, 20, 21] devoted to the calculation of the mass spectra of doubly heavy baryons in the three-body approach is a sum of the string potential V^{conf} and the Coulomb interaction potential V^C. A consistent derivation of the three-quark potential was done in the Wilson-loop approach in Ref.[27, 28] which accounts for corrections of order $1/m^2$. Accounting for the growth of the strong coupling constant for the interaction of a light quark with the heavy quark, we have included in the pure nonrelativistic three-quark Hamiltonian the vacuum polarization corrections of order α_s^2 in the form [29, 30]:

$$H_0 = \frac{\mathbf{p}_1^2}{2m_1}+\frac{\mathbf{p}_2^2}{2m_2}+\frac{\mathbf{p}_3^2}{2m_3}-\frac{2}{3}\sum_{i<j}\frac{\alpha_s^{ij}}{|\mathbf{r}_{ij}|}\left[1+\frac{\alpha_s^{ij}}{4\pi}\left(\tilde{a}_1+8\gamma_E\beta_0+8\beta_0\ln(\tilde{\mu}_{ij}|\mathbf{r}_{ij}|)\right)\right]+\sum_{i=1}^{3}\frac{1}{2}A(r_i+B). \tag{1}$$

We take the Hamiltonian (1) as the initial approximation in our study of the baryon mass spectrum. The operator of quark momenta $\mathbf{p}_i = -i\frac{\partial}{\partial \mathbf{r}_i}$, $\mathbf{r}_i$ is the position of quark i with respect to a common string-junction point which coincides approximately with the center-of-mass point (for a more detailed discussion see Refs.[19, 20, 21]) and $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ (compare with Fig.1). The parameters of the confinement part of the potential are the following [14, 17, 18]: $A = 0.18\ GeV^2$, $B = -0.16\ GeV$. We take the quark masses $m_b = 4.88\ GeV$, $m_c = 1.55\ GeV$, $m_{u,d} = 0.33\ GeV$, $m_s = 0.5\ GeV$ as in our previous [15, 16, 18] calculations of different hadron properties on the basis of the relativistic quark model. For the dependence of the QCD coupling constant $\alpha_s^{ij} = \alpha_s(\tilde{\mu}_{ij}^2)$ on the renormalization point $\tilde{\mu}_{ij}^2$ we use the two-loop result [31]

$$\alpha_s(\tilde{\mu}_{ij}^2) = \pi\left[\frac{1}{\beta_0 L}-\frac{\beta_1\ln L}{\beta_0(\beta_0 L)^2}\right],\ \beta_0=\frac{1}{4}\left(11-\frac{2}{3}n_f\right),\beta_1=\frac{1}{16}\left(102-\frac{38}{3}n_f\right),L=\ln(\tilde{\mu}_{ij}^2/\Lambda^2), \tag{2}$$

where $\Lambda = 0.168\ GeV$, $\tilde{\mu}_{ij} = 2m_im_j/(m_i+m_j)$ is the renormalization scale, $\tilde{a}_1 = (31 - 10n_f/3)/3$ and n_f is the number of flavours. The three-body Hamiltonian (1) and the Schrödinger equation can be reduced to the two-body form. For this aim, the three-body Jacobi coordinates become useful [19, 32, 33]:

$$\boldsymbol{\rho}_{ij} = \alpha_{ij}(\mathbf{r}_i - \mathbf{r}_j),\ \boldsymbol{\lambda}_{ij} = \beta_{ij}\left(\frac{m_i\mathbf{r}_i + m_j\mathbf{r}_j}{m_i+m_j} - \mathbf{r}_k\right), \tag{3}$$

where the coefficients α_{ij}, β_{ij} are expressed in terms of the appropriate reduced masses:

$$\alpha_{ij} = \sqrt{\frac{\mu_{ij}}{\mu}},\ \beta_{ij} = \sqrt{\frac{\mu_{ij,k}}{\mu}},\ \mu_{ij} = \frac{m_im_j}{(m_i+m_j)},\ \mu_{ij,k} = \frac{(m_i+m_j)m_k}{(m_i+m_j+m_k)}, \tag{4}$$

μ is an arbitrary mass parameter which disappears in final expressions. Evidently, the coordinate $\boldsymbol{\rho}_{ij}$ is proportional to the distance between quarks i and j, and the coordinate $\boldsymbol{\lambda}_{ij}$ is proportional to the distance between the quark k and the center-of-mass of quarks i, j. Together with the center-of-mass coordinate $\mathbf{R}_{c.m.}$ the Jacobi coordinates determine completely the position of the system.

In the center-of-mass frame ($\mathbf{R}_{c.m.} = 0$) the operator of the kinetic energy can be written in the Jacobi coordinates $\boldsymbol{\rho}, \boldsymbol{\lambda}$ as follows:

$$T_0 = -\frac{1}{2\mu}\left(\frac{\partial^2}{\partial \boldsymbol{\rho}^2} + \frac{\partial^2}{\partial \boldsymbol{\lambda}^2}\right) = -\frac{1}{2\mu}\left(\frac{\partial^2}{\partial R^2} + \frac{5}{R}\frac{\partial}{\partial R} + \frac{K^2(\Omega)}{R^2}\right), \tag{5}$$

where $K^2(\Omega)$ is the angular momentum operator, whose eigenfunctions (hyperspherical harmonics) are determined by the equation [34]:

$$K^2(\Omega) Y_K(\Omega) = -K(K+4) Y_K(\Omega). \tag{6}$$

Here Ω designates five angular coordinates and R is the six-dimensional hyperradius

$$R = \sqrt{\boldsymbol{\rho}_{ij}^2 + \boldsymbol{\lambda}_{ij}^2}, \quad \rho = R\cos\theta, \quad \lambda = R\sin\theta. \tag{7}$$

The baryon wave function $\Psi(\boldsymbol{\rho}, \boldsymbol{\lambda})$ can be presented as an expansion over functions $Y_K(\Omega)$:

$$\Psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) = \sum_K \Psi_K(R) Y_K(\Omega). \tag{8}$$

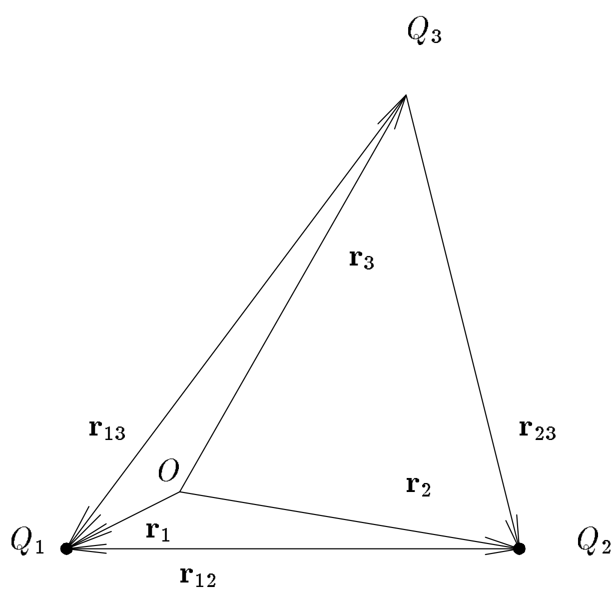

FIG. 1: The configuration of the three-quark system ($Q_1Q_2Q_3$). O is the string-junction point. Q_1, Q_2 are the heavy quarks b or c. The quark Q_3 is treated as a heavy quark b, c in the triply heavy baryon or a light quark q in the doubly heavy baryon.

Appearing here the radial wave functions $\Psi_K(R)$ satisfy to the system of differential equations [32, 33]. However, in the further study of the ground state triply and doubly

heavy baryons we use the hyperradial approximation in which $K=0$ and the bound state wave function $\Psi=\Psi(R)$ does not depend on the angular variables in the six-dimensional space. Using the Jacobi coordinates the Schrödinger equation for the three-quark system can now be transformed into the following form:

$$\left[-\frac{1}{2\mu}\left(\frac{d^2}{dR^2}+\frac{5}{R}\frac{d}{dR}\right)-\frac{2}{3}\sum_{i<j}\frac{\alpha_s^{ij}\alpha_{ij}}{|\boldsymbol{\rho}_{ij}|}+\sum_{i<j}\frac{1}{2}\left(A\gamma_{ij}|\boldsymbol{\lambda}_{ij}|+B\right)-\right. \tag{9}$$

$$\left.-\frac{1}{6\pi}\sum_{i<j}\frac{\alpha_s^{ij2}\alpha_{ij}}{|\boldsymbol{\rho}_{ij}|}\left(\tilde{a}_1+8\gamma_E\beta_0+8\beta_0\ln\frac{(\tilde{\mu}_{ij}|\boldsymbol{\rho}_{ij}|)}{\alpha_{ij}}\right)\right]\Psi(R)=E\Psi(R),$$

where coefficients γ_{ij} are given by

$$\gamma_{ij}=\sqrt{\frac{\mu(m_i+m_j)}{m_k(m_1+m_2+m_3)}}. \tag{10}$$

Averaging Eq.(9) over angular variables, we can present the Schrödinger equation for the reduced radial wave function $\chi(R)=R^{5/2}\Psi(R)$ in the form of a two-body wave equation:

$$\frac{d^2\chi(R)}{dR^2}+2\mu\left[E-V(R)\right]\chi(R)=0, \tag{11}$$

$$V(R)=\sum_{k=-2}^{1}a_kR^k+a_2\frac{\ln R}{R},\ a_0=\frac{3}{2}B,\ a_1=\frac{16}{15\pi}A\sum_{i<j}\gamma_{ij},\ a_{-2}=\frac{15}{8\mu}, \tag{12}$$

$$a_{-1}=-\frac{32}{9\pi}\sum_{i<j}\alpha_s^{ij}\alpha_{ij}\left[1+\frac{\alpha_s^{ij}}{\pi}\left(-\frac{17}{4}+\frac{9}{2}\ln 2+\frac{9}{2}\gamma_E+\frac{9}{2}\ln\frac{\tilde{\mu}_{ij}}{\alpha_{ij}}\right)\right],\ a_2=-\frac{16}{\pi^2}\sum_{i<j}\alpha_s^{ij2}\alpha_{ij}.$$

The Schrödinger equation (11) determines the baryon mass spectrum in the initial approximation. It can be solved numerically by the use the Mathematica program [35]. The baryon wave function $\chi(x)$ ($x=R\cdot\sqrt{\mu}$) for the state (ccc) is shown in Fig.2.

The next step in the solution of the spectral problem is related to the consideration of different corrections to the Hamiltonian H_0. In the present study we suggest that the three-quark potential has the form of the sum of twin quark the Breit-like interactions (we neglect orbital motion of the quarks) [15, 16, 17, 18]:

$$V^{SI}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=V^C(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)+V^{conf}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)+\sum_{k=1}^{6}\Delta V_k(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3), \tag{13}$$

$$\Delta V_1=-\sum_{i=1}^{3}\frac{A}{8m_i^2}\nabla_i^2 r_i+\sum_{i=1}^{3}\frac{A}{8m_i^2}\left\{r_i\left[\mathbf{p}_i^2-\frac{(\mathbf{p}_i\mathbf{r}_i)^2}{r_i^2}\right]\right\}_W+\sum_{i,j=1,i<j}^{3}\frac{A}{2m_im_j}\left\{r_{ij}\left[\mathbf{p}_{ij}^2-\frac{(\mathbf{p}_{ij}\mathbf{r}_{ij})^2}{r_{ij}^2}\right]\right\}_W, \tag{14}$$

$$\Delta V_2=\sum_{i,j=1,i\neq j}^{3}\frac{1}{8m_i^2}\nabla_i^2\left(-\frac{2}{3}\frac{\alpha_s^{ij}}{r_{ij}}\right), \tag{15}$$

$$\Delta V_3=-\sum_{i=1}^{3}\frac{B}{8m_i^2}\mathbf{p}_i^2-\sum_{i,j=1;i<j}^{3}\frac{B}{2m_im_j}\mathbf{p}_{ij}^2-\sum_{i=1}^{3}\frac{\mathbf{p}_i^4}{8m_i^3}, \tag{16}$$

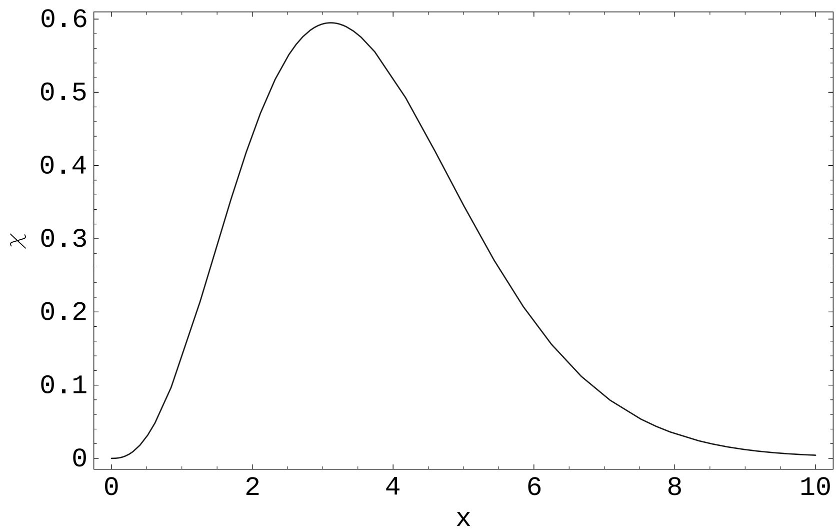

FIG. 2: The wave function of the baryon (ccc) obtained after numerical solution of the Schrödinger equation (11).

$$\Delta V_4 = \sum_{i,j=1;i<j}^{3} \frac{1}{2m_i m_j} \left\{ \frac{2}{3} \frac{\alpha_s^{ij}}{r_{ij}} \left[\mathbf{p}_i \mathbf{p}_j + \frac{(\mathbf{p}_i \mathbf{r}_{ij})(\mathbf{p}_j \mathbf{r}_{ij})}{r_{ij}^2} \right] \right\}_W , \tag{17}$$

$$\Delta V_5 = -\sum_{i,j=1;i<j}^{3} \frac{3\alpha_s^{ij\,2}}{8\pi m_i^2} \nabla_i^2 \frac{\ln(\tilde{\mu}_{ij} r_{ij})}{r_{ij}}, \tag{18}$$

$$\Delta V_6 = \sum_{i,j=1;i<j}^{3} \frac{3\alpha_s^{ij\,2}}{2\pi m_i m_j} \left\{ \mathbf{p}_i \mathbf{p}_j \frac{\ln(\tilde{\mu}_{ij} r_{ij})}{r_{ij}} + \frac{(\mathbf{p}_i \mathbf{r}_{ij})(\mathbf{p}_j \mathbf{r}_{ij})}{r_{ij}^2} \left(\frac{\ln(\tilde{\mu}_{ij} r_{ij})}{r_{ij}} - \frac{1}{r_{ij}} \right) \right\}_W , \tag{19}$$

where $\{...\}_W$ denotes the Weyl ordering of operators. Accounting that the hyperradius R is independent of the order of the quark numbering we can average the potentials (14)-(19) over the functions $\Psi(R)$ to find their contributions to the baryon mass. The basic relation used for this aim is the following:

$$\Delta E_k = \int \frac{d\boldsymbol{\rho} d\boldsymbol{\lambda}}{\pi^3} \Psi(R) \Delta V_k(\boldsymbol{\rho}, \boldsymbol{\lambda}) \Psi(R). \tag{20}$$

Let us note, that the volume element $d\boldsymbol{\rho} d\boldsymbol{\lambda}$ can be written in terms of hyperradius R and the angle θ ($\rho = R\cos\theta$, $\lambda = R\sin\theta$, $0 \leq \theta \leq \pi/2$):

$$d\boldsymbol{\rho} d\boldsymbol{\lambda} = (4\pi)^2 R^5 \sin^2\theta \cos^2\theta dR d\theta. \tag{21}$$

Having the solution of Eq.(11) in the numerical form, we have calculated all corrections from the interaction operators (14)-(19) numerically. In Table I we present the values of corresponding spin-independent matrix elements for the three-quark system ($Q_1Q_2Q_3$) with the precision 0.001 GeV. This is not the accuracy of the mass spectrum calculation because the different theoretical uncertainties remain large (see the discussion below). A number of the Breit-like potentials (14)-(19) is dependent on the relative quark distances $\mathbf{r}_{ij}$ which could be expressed directly through the variables $\boldsymbol{\rho}_{ij}$: $\mathbf{r}_{ij} = \boldsymbol{\rho}_{ij}/\alpha_{ij}$. The confinement contributions and the pure relativistic corrections $\sum_{i=1}^{3} \mathbf{p}_i^4/8m_i^3$ in the potential (13) are determined in

TABLE I: Contributions of the spin-independent terms of the potential (13) to the mass spectrum of the triply and doubly heavy baryons (in GeV).

Quark content	$<\Delta V_1>$	$<\Delta V_2>$	$<\Delta V_3>$	$<\Delta V_4>$	$<\Delta V_5>$	$<\Delta V_6>$	Summary contribution
(ccc)	0.032	0.010	-0.080	-0.019	-0.001	0.001	-0.057
(ccb)	0.016	0.009	-0.059	-0.012	-0.002	0.0001	-0.048
(bbc)	0.007	0.007	-0.033	-0.008	-0.002	-0.0003	-0.029
(bbb)	0.005	0.003	-0.018	-0.006	-0.001	-0.001	-0.018
(ccq)	0.037	0.044	-0.243	-0.037	0.020	0.013	-0.166
(ccs)	0.046	0.025	-0.167	-0.029	0.004	0.006	-0.115
(bbq)	-0.035	0.039	-0.131	-0.012	0.013	0.003	-0.123
(bbs)	-0.009	0.022	-0.083	-0.010	0.001	0.001	-0.078
(bcq)	-0.002	0.042	-0.214	-0.023	0.016	0.008	-0.173
(bcs)	0.015	0.024	-0.143	-0.018	0.002	0.003	-0.117

terms of the variables $\boldsymbol{\lambda}_{ij}$: $\mathbf{r}_k = -\gamma_{ij}\boldsymbol{\lambda}_{ij}$. The perturbative and nonperturbative relativistic corrections are grouped together for the convenience in the expression ΔV_3. The sign of the first term in the potential (14) differs from the corresponding expression in Ref.[27]. The reason consists in the values of the universal Pauli interaction constant $\kappa = -1$ and the mixing coefficient $\epsilon = -1$ in the relativistic quark model fixed from the analysis of heavy quarkonium masses and radiative decays.

Another part of the total Hamiltonian depends on the quark spins. Neglecting the quark orbital momentum we present the spin-dependent part of the potential in the simple form [18]:

$$V^{SD}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{i<j} c_{ij}\mathbf{S}_i\mathbf{S}_j\delta(|\mathbf{r}_{ij}|), \tag{22}$$

$$c_{ij} = \frac{16\pi\alpha_s^{ij}}{9m_im_j}\left[1 + \frac{\alpha_s^{ij}}{\pi}\left(\frac{5}{3}\beta_0 - \frac{11}{3} - \left(\frac{m_i - m_j}{m_i + m_j} + \frac{1}{8}\frac{m_i + m_j}{m_i - m_j}\right)\ln\frac{m_j}{m_i}\right)\right]. \tag{23}$$

We assume that the color-magnetic interaction (22) can also be treated perturbatively as potentials (14)-(19). The matrix element of the operator (22) can be transformed as follows:

$$\Delta E^{SD} = \frac{64}{9}\sum_{i<j}\frac{\alpha_s^{ij} < \mathbf{S}_i\mathbf{S}_j > \alpha_{ij}^3}{m_im_j\pi}\int_0^\infty \lambda^2 d\lambda |\Psi(\lambda)|^2 \times \tag{24}$$

$$\times\left[1 + \frac{\alpha_s^{ij}}{\pi}\left(\frac{5}{3}\beta_0 - \frac{11}{3} - \left(\frac{m_i - m_j}{m_i + m_j} + \frac{1}{8}\frac{m_i + m_j}{m_i - m_j}\right)\ln\frac{m_j}{m_i}\right)\right].$$

To obtain the contribution of the spin-dependent interaction (22) to the energy spectrum we have to perform the addition of the quark spins $\mathbf{S}_i$ to the baryon spin $\mathbf{S}$. Designating the heavy quark spins $\mathbf{S}_1$ and $\mathbf{S}_2$, we introduce the spin of a doubly heavy diquark $\mathbf{S}_{12} = \mathbf{S}_1 + \mathbf{S}_2$. Then, the baryon spin is $\mathbf{S} = \mathbf{S}_{12} + \mathbf{S}_3$. Considering that the baryon wave function $\Psi_{S_{12}SS_z}$ is the eigenfuction for the operators $\mathbf{S}_{12}^2$, $\mathbf{S}^2$, S_z, we can express the scalar product $(\mathbf{S}_1\mathbf{S}_2)$

TABLE II: Mass spectrum of ground states of triply heavy baryons (in GeV). $\{Q_1Q_2\}$ denotes the axial vector two-quark combination (diquark). The hyperfine splitting has been neglected in Refs.[9, 10, 11].

Baryon	Quark content	J^P	This work	[9]	[10]	[11]
Ω_{ccc}	(ccc)	$3/2^+$	4.803	4.79	4.925	4.76
Ω_{ccb}	$\{cc\}b$	$1/2^+$	8.018	—	—	—
Ω^*_{ccb}	$\{cc\}b$	$3/2^+$	8.025	8.03	8.200	7.98
Ω_{bbc}	$\{bb\}c$	$1/2^+$	11.280	—	—	—
Ω^*_{bbc}	$\{bb\}c$	$3/2^+$	11.287	11.20	11.480	11.19
Ω_{bbb}	(bbb)	$3/2^+$	14.569	14.30	14.760	14.37

TABLE III: Mass spectrum of ground states of doubly heavy baryons (in GeV). $\{Q_1Q_2\}$ denotes the axial vector two-quark combination (diquark), $[Q_1Q_2]$ denotes the scalar one.

Baryon	Quark content	J^P	This work	[15]	[22]	[23]	[4]	[21]
Ξ_{cc}	$\{cc\}q$	$1/2^+$	3.510	3.620	3.478	3.66	3.61	3.69
Ξ^*_{cc}	$\{cc\}q$	$3/2^+$	3.548	3.727	3.61	3.74	3.68	
Ω_{cc}	$\{cc\}s$	$1/2^+$	3.719	3.778	3.59	3.74	3.71	3.86
Ω^*_{cc}	$\{cc\}s$	$3/2^+$	3.746	3.872	3.69	3.82	3.76	
Ξ_{bb}	$\{bb\}q$	$1/2^+$	10.130	10.202	10.093	10.34		10.16
Ξ^*_{bb}	$\{bb\}q$	$3/2^+$	10.144	10.237	10.133	10.37		
Ω_{bb}	$\{bb\}s$	$1/2^+$	10.422	10.359	10.18	10.37		10.34
Ω^*_{bb}	$\{bb\}s$	$3/2^+$	10.432	10.389	10.20	10.40		
Ξ_{cb}	$\{cb\}q$	$1/2^+$	6.792	6.933	6.82	7.04		6.96
Ξ'_{cb}	$[cb]q$	$1/2^+$	6.825	6.963	6.85	6.99		
Ξ^*_{cb}	$\{cb\}q$	$3/2^+$	6.827	6.980	6.90	7.06		
Ω_{cb}	$\{cb\}s$	$1/2^+$	6.999	7.088	6.91	7.09		7.13
Ω'_{cb}	$[cb]s$	$1/2^+$	7.022	7.116	6.93	7.06		
Ω^*_{cb}	$\{cb\}s$	$3/2^+$	7.024	7.130	6.99	7.12		

through the diquark spin $\mathbf{S}_{12}$:

$$\mathbf{S}_1\mathbf{S}_2 = \frac{1}{2}\left[S_{12}(S_{12}+1) - \frac{3}{2}\right], \tag{25}$$

where $S_{12} = 0$ (the scalar diquark) or $S_{12} = 1$ (the vector diquark). For averaging the two other spin dependent terms $(\mathbf{S}_1\mathbf{S}_3)$ and $(\mathbf{S}_2\mathbf{S}_3)$ there is a need to use the unitary transformation from the wave functions $\Psi_{S_{12}SS_z}$ to the eigenfunctions $\Psi_{S_{13}SS_z}$ (or $\Psi_{S_{23}SS_z}$) [36]:

$$\Psi_{S_{12}SS_z} = \sum_{S_{13}} (-1)^{S_1+S_2+S_3+S} \sqrt{(2S_{12}+1)(2S_{13}+1)} \begin{Bmatrix} S_3 & S_1 & S_{13} \\ S_2 & S & S_{12} \end{Bmatrix} \Psi_{S_{13}SS_z}. \tag{26}$$

Corresponding values of $6j$ - symbols are taken from Ref.[36]. The presence of operators $(\mathbf{S}_1\mathbf{S}_3)$ and $(\mathbf{S}_2\mathbf{S}_3)$ leads to a mixing between states with different spin S_{12} of the doubly heavy diquark and definite value $S = 1/2$. In the case of the (bcq) and (bcs) baryons we have the following mixing matrices:

$$\begin{pmatrix} -0.005 & 0.015 \\ 0.015 & -0.020 \end{pmatrix}[GeV], \quad \begin{pmatrix} -0.004 & 0.010 \\ 0.010 & -0.015 \end{pmatrix}[GeV]. \tag{27}$$

After the matrix diagonalization we obtain the numerical values for the masses of the baryons (bcq) and (bcs) $J^P = 1/2^+, 3/2^+$ which are presented in Table III. In the case of triply and doubly heavy baryons with two identical heavy quarks c or b, the diquarks (cc) and (bb) have the spin 1 (the axial vector diquark). The hyperfine mass splittings for doubly heavy (cc) and (bb) baryons are the following: $\Delta M(\Xi_{cc}) = 0.038$ GeV, $\Delta M(\Omega_{cc}) = 0.027$ GeV, $\Delta M(\Xi_{bb}) = 0.014$ GeV, $\Delta M(\Omega_{bb}) = 0.010$ GeV. They are more than two times smaller as compared with the splittings obtained in Ref.[17]. Whereas the masses of triply heavy baryons are in the agreement with the earlier performed calculations in Refs.[9, 10, 11] the obtained masses of doubly heavy baryons lie for the most part lower than our predictions from [17]. We expect that higher order corrections both in α_s and $1/m$ could change these results.

Special attention has to be given to the accuracy of the performed calculation. In the case of triply heavy baryons the expansion over $|\mathbf{p}_{b,c}|/m_{b,c}$ is well defined. Numerical estimate of relativistic effects gives the following expectation values: $< \mathbf{p}_c^2/m_c^2 > \approx 0.34$, $< \mathbf{p}_b^2/m_b^2 > \approx 0.09$. So, the next to leading order contribution $\mathbf{p}^4/m^4$ leads to the theoretical uncertainty ± 0.030 GeV for the charm baryons and ± 0.005 GeV for the bottom baryons (see Table II). The presence of the light quark in the baryon leads to the increase of the contribution of the relativistic effects connected with the motion of quarks q and s to the mass spectrum. Indeed, we can estimate the average value of the square momentum of light quark using the obtained baryon wave function (see Fig.2). Introducing the parameter $< \mathbf{p}_3^2/m_3^2 >$ where the brackets designate the matrix element with the functions of Eq.(9) we find that in the case of light quark q it has the value of order unity. So, summary perturbative and nonperturbative relativistic contribution is sufficiently large (see third column of Table I). Nevertheless, we suppose that three terms of the expansion $\epsilon_3 = \sqrt{\mathbf{p}_3^2 + m_3^2} \approx m_3 + \mathbf{p}_3^2/2m_3 - \mathbf{p}_3^4/8m_3^3$ represent the light quark relativistic energy ϵ_3 with sufficiently high accuracy (nearly 10%). The most significant error is connected with the relativistic corrections in the second order perturbation theory (PT). The reduced Green's function $G(\mathbf{R}, \mathbf{R}')$ for Eq.(11) can be constructed numerically using the solutions of the Schrödinger equation (11) for different principal quantum numbers n. Saturating the sum over n in the expression of the Green's function by 10-15 excited states, we obtain approximate value $G(\mathbf{R}, \mathbf{R}')$ which can be employed in the second order PT. Our preliminary estimate of the relativistic contributions in the second order PT shows that these corrections can comprise near 30% of the contribution $< \Delta V_3 >$ (see Table I).

Let us note that exists another possibility to rationalize the relativistic energy of the light quark. It demands the introduction of the effective energy (the momentum squared) of the light quark in the bound state. Indeed, we can use the following transformation of the light quark energy: $\epsilon_3 = \frac{\mathbf{p}_3^2 + m_3^2}{\epsilon_3} \to \frac{\mathbf{p}_3^2 + m_3^2}{E_3} = \frac{\mathbf{p}_3^2}{2\tilde m_3} + \frac{m_3^2}{E_3}$, where $\tilde m_3 = E_3/2$, and the light quark effective energy E_3 can be expressed in terms of the baryon mass: $E_3 = M_B - m_1 - m_2$. In this case the quantity $\tilde m_3$ plays the role of new effective mass of the light quark in the baryon and the addendum $\tilde m_3^2/E_3$ is equivalent to the term $m_3/2$ used in Refs.[19, 20, 21] if it is granted

that the effective mass $\tilde{m}_3$ coincides with the composite quark mass m_3. Numerical value of the light quark effective energy $E_3 \approx 0.4 \div 0.5\, GeV$, and the shift $m_3^2/E_3 \approx 0.21 \div 0.22\, GeV$ is in the agreement with the value of the term $m_3/2$ in Refs.[19, 20, 21]. The appearance of the effective quantities in this approach evidently leads to the definite theoretical uncertainty in the baryon mass calculation. So, the presence of the light quark in the doubly heavy baryon essentially complicates the mass spectrum calculation with the sufficiently high accuracy. To improve the theoretical results of Table III the corrections of the second order PT should be calculated together with the matrix elements of the potential up to the $1/m^4$ order. The ground state wave function of the triply charm or bottom baryons is evidently the angle independent. The used hyperradial approximation is valid for it with high accuracy. For triply heavy baryons (ccb), (bbc) or for doubly heavy baryons (Q_1Q_2q) this hyperradial approximation is less applicable. So, the dependence of the baryon wave function on the angular variables in the six-dimensional space also has to be taken into account.

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